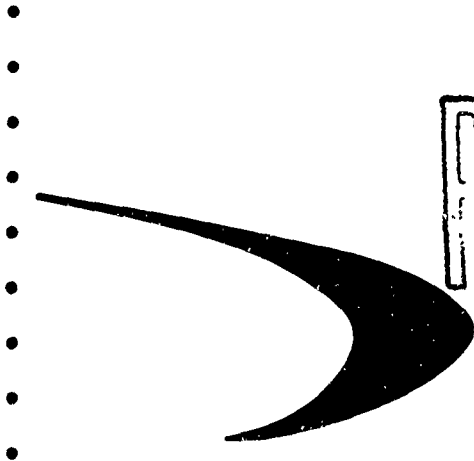
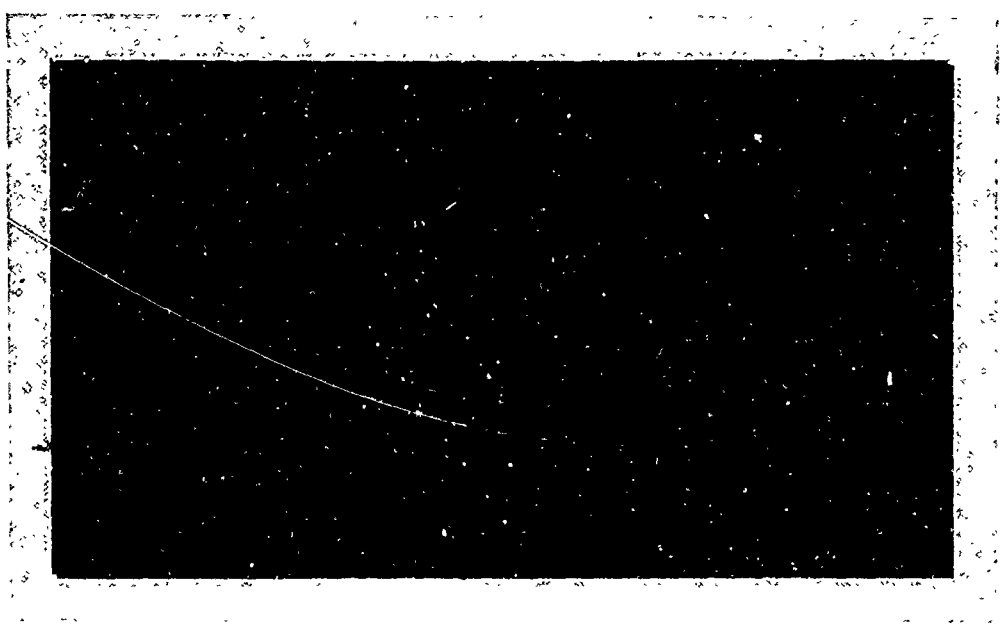


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Report No. 125-9

SOLUTION OF BLAST WAVES BY A CONSTANT TIME SCHEME
IN THE METHOD OF CHARACTERISTICS

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TABLE OF CONTENTS

ABSTRACT.....	iii
LIST OF FIGURES.....	iv
NOMENCLATURE.....	v
INTRODUCTION.....	1
FORMULATION OF THE PROBLEM FOR NUMERICAL CALCULATION.....	2
NUMERICAL CALCULATION.....	3
EXTRAPOLATION OF NUMERICAL RESULTS.....	6
DOMAIN OF DEPENDENCE AND RANGE OF INFLUENCE.....	7
CONCLUSION.....	9
REFERENCES.....	10
FIGURES.....	11
APPENDIX A.....	18

ABSTRACT

The numerical method of characteristics with constant time grids is applied to the plane blast wave problem. The calculation begins at a constant time line bounded by the shock front and a fixed back boundary which lies halfway between the shock front and the time axis. Along the initial time line, all three flow variables are prescribed and on the back boundary, the local flow velocity is prescribed. The prescribed values are calculated from the similarity solution. For the shock front, the strong shock equations are applied.

When compared to the exact similarity solution, the results of this method are found to be accurate to within .5% for all variables throughout the region of calculation, after a pressure drop of 98%. Both h^2 -type and h -type extrapolations of calculated results improve the accuracy; however, h -type extrapolation is more favorable.

The effects on the domain of dependence and range of influence are studied briefly.

LIST OF FIGURES

1. Physical Plane Showing Region of Numerical Solution for Blast Wave Problems.....	11
2. Particle Velocity, Sound Speed, and Pressure Distribution Within the Blast Wave.....	12
3. Plane Wave Error Analysis.....	13
4. Error on Shock Front.....	14
5. Curves of Relative Error in Pressure for the Plane Blast Wave.....	15
6. Curves of Relative Error in Pressure for the Plane Blast Wave.....	16
7. Physical Plane Showing Widening of Range of Influence and Relative Errors.....	17

NOMENCLATURE

- c = sound speed
- E = parameter proportional to the total energy within the wave
- h = parameter proportional to the mesh size
- k = x_b/x_s position ratio of back boundary
- p = pressure
- t = time variable
- u = particle velocity
- U = shock wave velocity
- x = space variable
- γ = specific heat ratio
- ρ = density
- ρ_1 = constant density outside wave zone

Subscripts

- s = at the shock front
- b = at the back boundary

INTRODUCTION

This report is a continuation of the report "Solution of Blast Wave by the Method of Characteristics."¹ To avoid unnecessary repetition, the background in the choice of the particular problem, the governing equations, and the similarity solution of the blast waves are not included in this report.

In applying the numerical method of characteristics, two techniques are available; namely, the standard technique of following the natural characteristic grid and the technique of using constant time increments. In the standard technique, as developed in Ref. 1, the points of intersection of the two families of characteristics do not occur at equal intervals in either the space or time coordinates. When the spatial distribution of the flow variables at a later instant is required, we have to perform a two-dimensional interpolation, which is not always desirable.

In the present report, the constant time technique will be studied. This technique, originally proposed by Hartree², overcomes the difficulty of uneven distribution of grid points by choosing grid points on pre-determined constant time lines. This has the advantage that the required interpolation is always one-dimensional. Also, the present method gives results along particle paths in successive steps. Therefore, it is advantageous in dealing with problems where interfaces are involved.

Calculations are performed for the plane blast wave problem using the constant time method, and the results are compared with the exact similarity solution.³ Within the region of calculation, indicated in Fig. 3a, the results were found to be accurate to within .5%. The maximum pressure at the last time line is about 2% of its original value.

Three sets of calculations, each with a different mesh size, were made for the same problem. The results were extrapolated with h -type and h^2 -type extrapolation formulas. Both extrapolation results improve the accuracy of the results. The h -type extrapolation gives a better result.

It is felt that the constant time method will introduce error due to broadening of the domain of dependence. Brief investigation indicated that this is true; however, the error is relatively small.

FORMULATION OF THE PROBLEM FOR NUMERICAL CALCULATION

The one-dimensional blast wave solution is one of the few exact solutions for the non-isentropic (entropy changes from particle to particle), unsteady flow. This type of flow will be calculated by the numerical method (constant time) and the results compared with the closed form similarity solution.

The region in the physical plane to be calculated is shown in Fig. 1. On the bottom it is bounded by the constant time initial line. Along this line, $t = t_0$, values of all the three dependent variables u , c , and p are specified. In applying the constant time method it is desirable to avoid the origin ($x = 0$, and $t = 0$) and the time axis. The origin is a singular point with unbounded values of pressure, particle velocity, sound speed, and along the time axis, the sound speed is infinite. For reasons to be given subsequently the time increments, Δt , between successive constant time lines are so chosen that they are inversely proportional to the local sound velocity; in regions with extremely high sound speed Δt approaches zero, making the method impractical. Hence, a back boundary

with the equation $x_b = (1/2)x_s$ is introduced on the left. Here, x_b and x_s are the corresponding x coordinates of the back boundary and shock front, respectively. According to the terminology used by Courant and Friedrichs,⁴ the back boundary is a "time like" arc, since two of the three characteristics, II and III, reach this curve from the interior of the region R . The correct boundary condition requires that one of the three dependent variables be specified on the back boundary. In this problem, the particle velocity u was calculated from the exact solution and used as the one required boundary condition. The right boundary of the region of numerical solution is formed by the shock front. At a point on the shock front, there are four unknown quantities to be determined, i.e., u , p , c , and U . The three strong shock conditions together with the equation of the I-characteristic serve to determine the four unknowns. The path of the shock front may be established by integrating the shock velocity U .

NUMERICAL CALCULATIONS

Along the initial time line, $t = t_0$, a number of equally spaced points are chosen, and the properties, u , c , and p at these points are calculated from the exact solution. Then a new time line can be drawn with a time increment Δt . The magnitude of the time increment is limited by the condition $\Delta t < \Delta x/c$, where Δx is the spatial interval between any two neighboring points on the initial time line and c is the corresponding local velocity of sound. With this criterion, the I and II characteristics from a point on the $t = t_0 + \Delta t$ line will not encompass more than one mesh point on the $t = t_0$ line. Draw III

characteristics (path line) from each of the initial points. The intersections of these path lines and the new time line establish a set of new points about which the flow properties are to be evaluated. To accomplish this, the state and physical characteristic equations have to be solved. Since the equations for physical characteristics involve the state variables u and c and the state characteristic equations contain the physical variable t , neither of them can be solved independently. Numerical iterative schemes are devised to obtain approximate solutions and the detailed iterative procedures are described in Appendix A. Referring to Fig. A.2, the iteration procedure involves establishing the III-characteristic through an initial point 2, locating the intersection, i.e., point 4, of the particle path and the constant time line $t = t_0 + \Delta t$, tracing back the I and II-characteristics from point 4 until they reach the initial time line at points A and B. The properties at A and B are calculated by interpolation of values at points 1, 2, and 3. The fact that the entropy along a particle path remains constant is used in relating properties at point 2 and point 4.

The form of the interpolation formulas used in obtaining properties at points A and B are highly influential to the accuracy of the constant time method. Linear interpolation would be the simplest in computation. Unfortunately, as shown in Fig. 2, the pressure p near the shock front and the sound velocity close to the back boundary are both highly non-linear with respect to the spatial coordinate x . Linear interpolation would cause appreciable error for the values of these properties. Furthermore, the errors tend to accumulate with time. Errors due to linear interpolation which might be tolerable in usual cases are

objectionable in the present problem. For this reason quadratic interpolation formulas were used in interpolating p and c while linear interpolation was used for u . Local velocity u is essentially linear with respect to x throughout the region of solution R .

At the back boundary and the shock front modified iteration procedures are adopted. Since the shock front propagates outward engulfing new particles continuously and in the mean time flow particles leave the region of solution across the back boundary, sub-routines are provided to terminate a particle path which is about to leave and to initiate a new particle path near the shock front when the distance between the shock front and the closest data point equal or larger than the local Δx . Therefore, the number of data points between successive time lines varies. In the present case, the total number of data points along a constant time line increases slightly with time. This means there are less particle paths passing through the back boundary than the new particle paths initiated at the shock front. The rate of increase of data points is slow; which is not enough to cause concern about consuming too much computation time.

The numerical procedure was programmed in Fortran II language and the computation was done on an IBM 7040 computer. The computation time for the case of 51 initial data points, $h = 1/2$, was approximately half an hour, which gave a pressure drop of 98% on the shock front. The accuracy of this calculation is shown in Fig. 3. Figure 3a shows the physical plane, indicating the position of the shock front, the back boundary, and the last line of numerical calculation. Figure 3b shows the relative errors of p , u , and c at points on the last line of the calculation as compared to the exact solution. The relative errors of t , p , u , and c along the shock front are shown in Fig. 3c. The maximum

error for all variables throughout the entire region of calculation is less than .5%.

EXTRAPOLATION OF NUMERICAL RESULTS

In order to study what type of extrapolation, h -type or h^2 -type (Ref. 1), may improve the numerical results, the same plane blast wave problem was calculated three times using three different numbers of initial data points. Identical computations were performed with 26, 51, and 101 initial points which correspond to mesh sizes $h = 1$, $1/2$, and $1/4$, respectively. The relative errors in p , u , c , and t along the shock front for the three cases are plotted in Fig. 4. The curve labeled $h = 1/2$ represents the error of the indicated properties with mesh size $h = 1/2$. The errors decrease as the mesh size decreases and the general shape of these error curves resemble each other within their own group. The highest error appears in p ; while the least in u and c . The extrapolated values of p along the shock front were compared with the exact solutions. Results are presented in Fig. 5 and Fig. 6. In these figures the original errors were also plotted for easy comparison. The curve labeled $h^2(1, 1/2)$ represents the error in the extrapolated value of p calculated from the h^2 -type formula, by using the pressures from corresponding calculations with $h = 1$ and $h = 1/2$; $h(1, 1/2, 1/4)$ represents h -type extrapolation of pressures corresponding to $h = 1$, $1/2$, and $1/4$, etc.

These figures indicate that:

1. The extrapolated results from any two sets of calculations are always more accurate. This is true for both h-type and h^2 -type extrapolation.
2. Results obtained from the h-type extrapolation are more accurate than those from the h^2 -type extrapolation.
3. Extrapolation from three sets of calculated results are slightly better than those from two sets of calculations.

DOMAIN OF DEPENDENCE AND RANGE OF INFLUENCE

It is known from theory of partial differential equations of hyperbolic type that the domain of dependence of a point in the physical plane lies on the portion of the initial line that is bounded by the I and II characteristics from the point. For instance, any change in flow properties outside the domain \overline{AB} in Fig. A.2, certainly will not effect the properties at point 4 theoretically. By using the constant time method, properties at points A and B are determined by interpolating properties at points 1, 2, and 3. Consequently, any change of values of properties at points 1 and 3 will in fact change the values of properties at points A and B, and in turn will change the values of the properties at point 4. Apparently this is an error attributed to the numerical method. The criterion $\Delta t = R(\Delta x/c)$, where $R < 1$, used in the choice of Δt is equivalent to requiring that points A and B lie inside of point 1 and 3. The broadening of the domain of dependence from \overline{AB} to $\overline{13}$ can also be viewed as the widening of the range of influence of a point on the initial time line.

To evaluate the error introduced by this broadening of the domain of dependence, several calculations were made for the same problem, with computations confined in a small region R in Fig. 7a. The prescribed properties for the equally spaced points on the initial line were calculated from the exact solution. The method of characteristics was first used in the calculation to determine the II-characteristic passing through point L. This established a boundary, shown in Fig. 7b. To the left of this boundary, the flow properties should not be influenced by any changes in properties at and to the right of point L. Then four sets of calculations were performed using the constant time method. The first set to be used as a standard for comparison is calculated from correct prescribed initial values. In the other three calculations the flow properties at point L were changed. The changes were made by varying one of the three properties, u , c , or p , and keeping the other two unchanged. For example, in the calculation involving a change in p , its value was reduced by 5% at point L. The deviation of the pressure from the first set of calculation in the region to the left of LP were calculated and indicated in Fig. 7b. Similar results for u and c may also be obtained. Such deviations are deemed as errors due to the constant time method. It should be mentioned here that the time increments in all the four sets of calculations were the same but the x coordinates of the data points do not coincide exactly between different sets of calculations. In computing the relative errors, such deviations in x coordinates were compensated through linear interpolation. The range of influence of point L is widened and it is bounded on the left by LQ which has a slope

of $(dx/dt) = u - (c/R)$ rather than $(dx/dt)_{II} = u - c$. Since R is always less than one, the dotted line will therefore be situated to the left of the II-characteristic, LP. However, the error is comparatively small and tends to diminish gradually with time.

CONCLUSION

The numerical method of characteristics with constant time grids produces very accurate results when applied to plane blast wave problem, provided the region with extremely high sound velocity is excluded. For other one-dimensional unsteady fluid flow problems which do not include regions with extremely high velocity of sound, it is reasonable to believe that this numerical method will also yield accurate results.

Both h -type and h^2 -type extrapolation of the numerical results improve the accuracy, whereas h -type extrapolation gives a better result. The accuracy of the numerical results is sensitive to the interpolation formula employed. This method is particularly useful for problems involving interfaces which need to be traced.

Although the results of the plane blast wave problem only are reported, the same method with slight modification can be used to solve problems with cylindrical and spherical symmetry. Work is being done in this area and will be reported at a later date.

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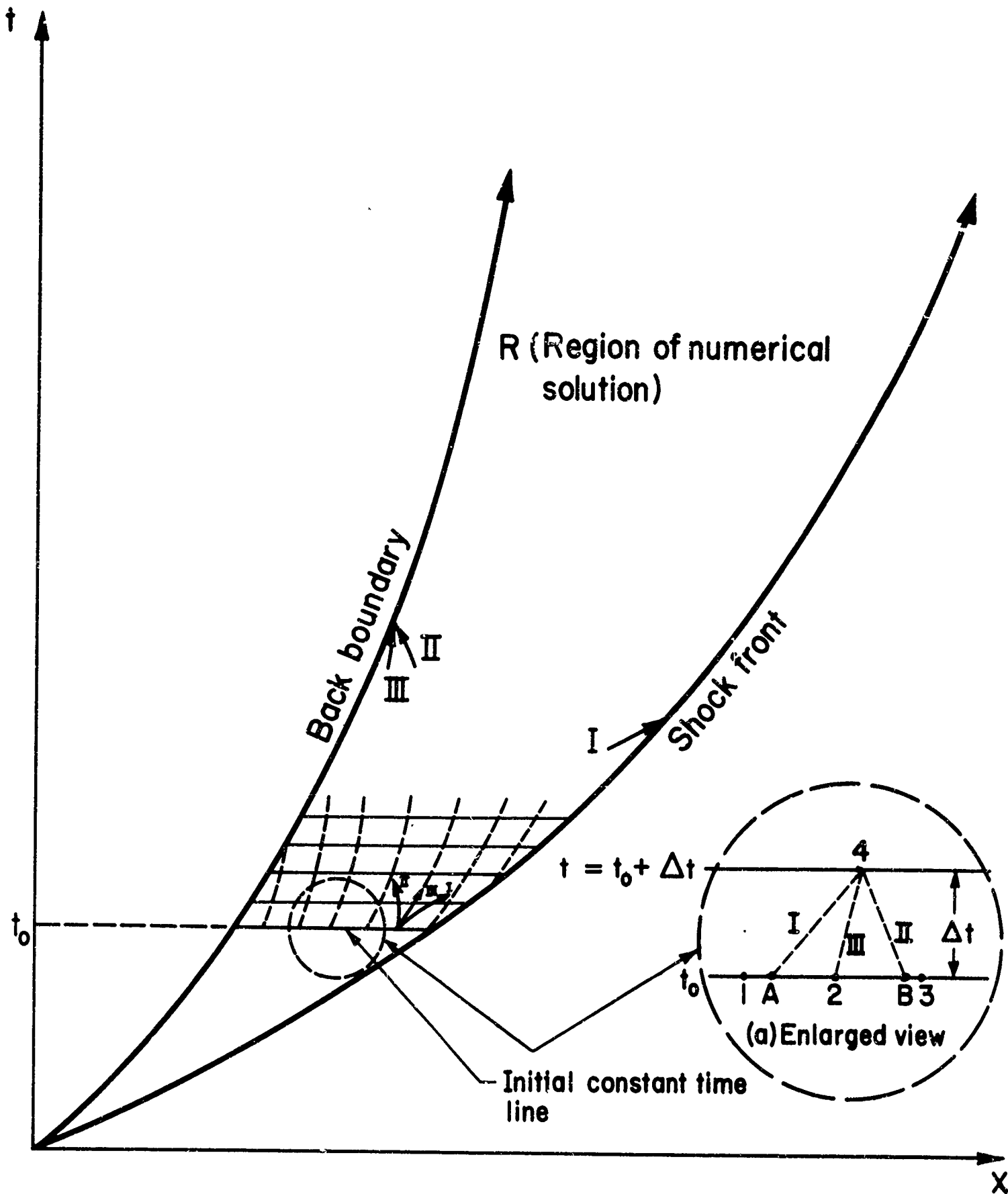


Figure 1 Physical Plane Showing Region of Numerical Solution for Blast Wave Problems

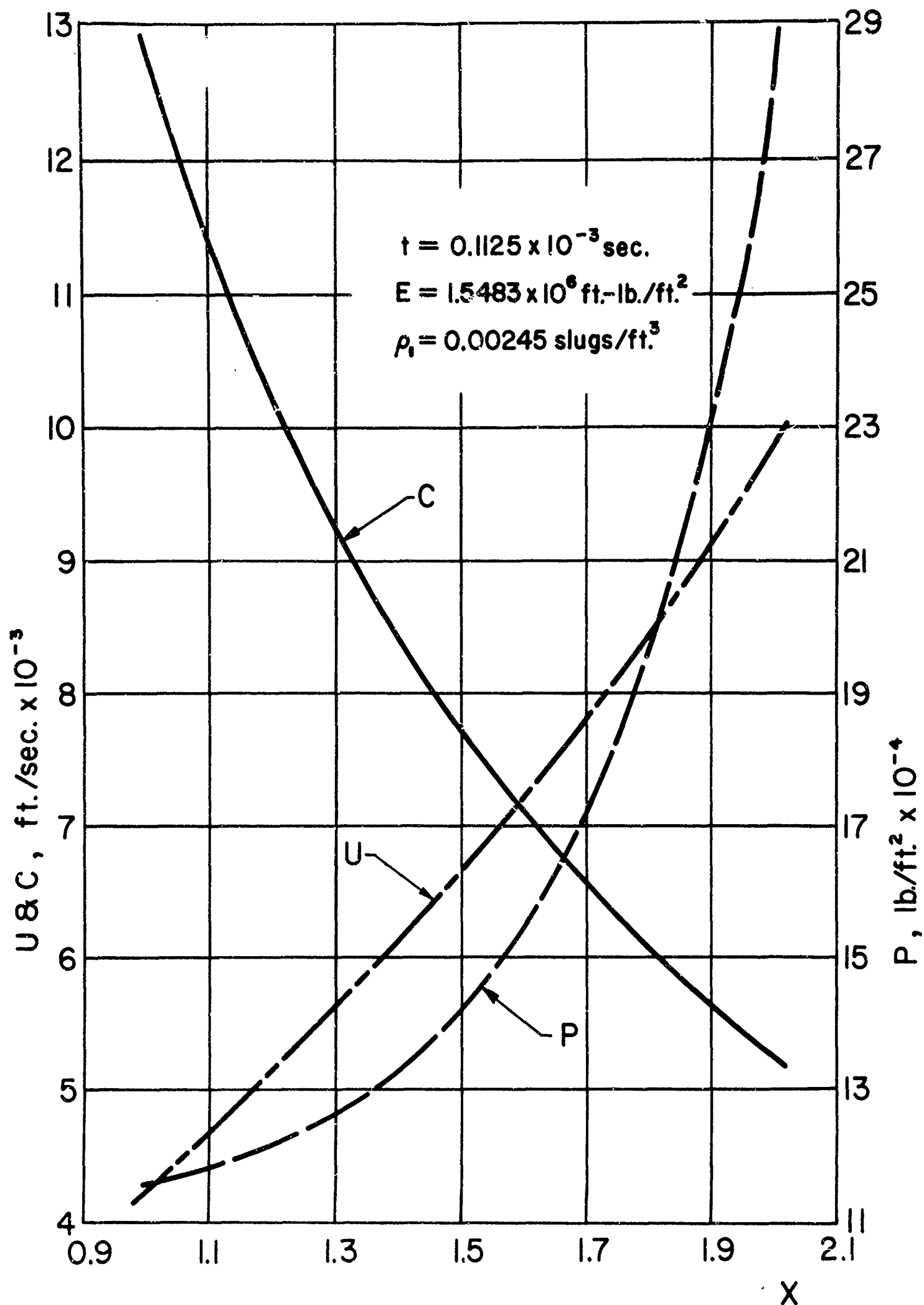


Figure 2 Particle Velocity, Sound Speed, and Pressure Distribution within the Blast Wave

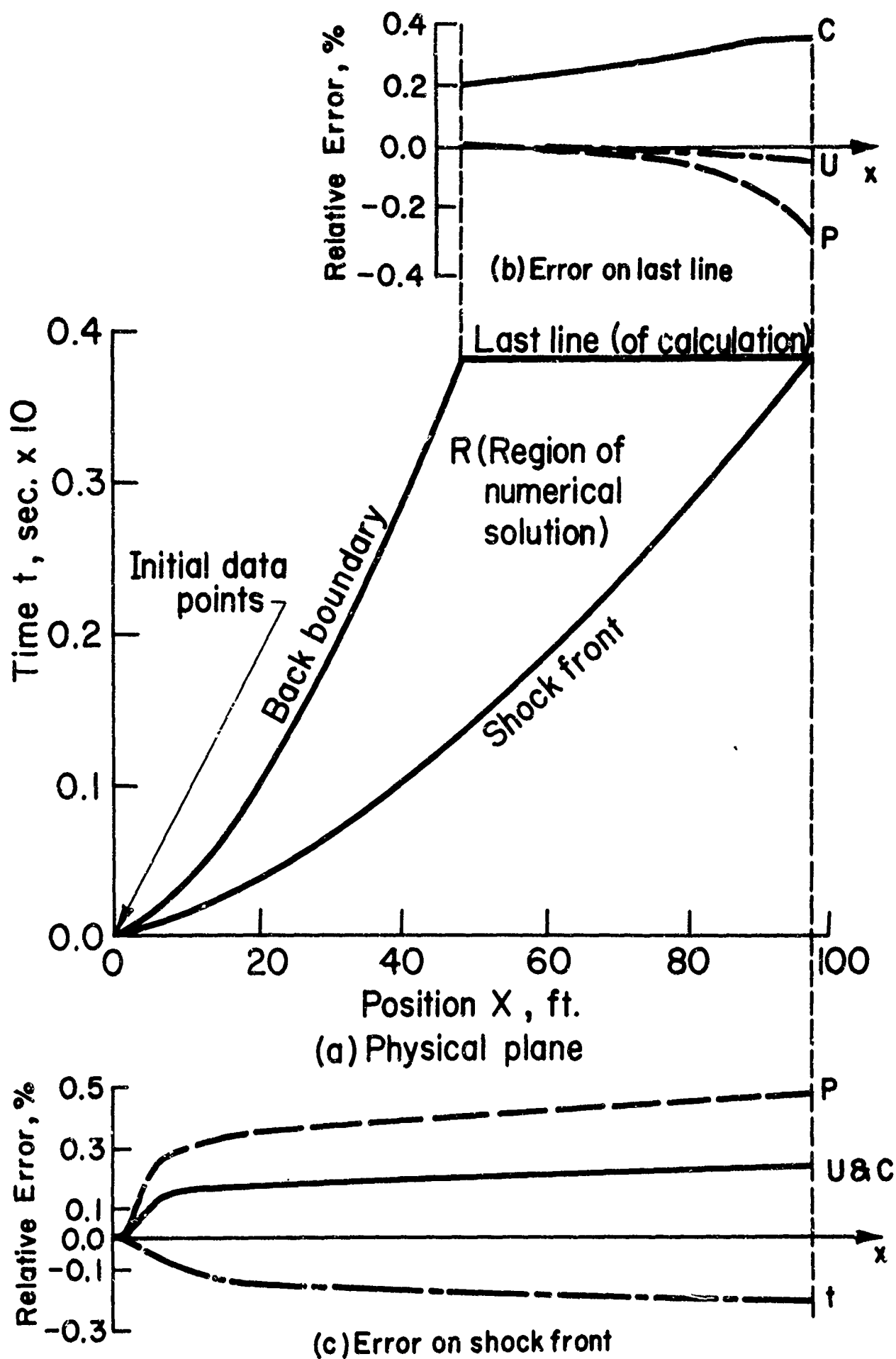


Figure 3 Plane Wave Error Analysis

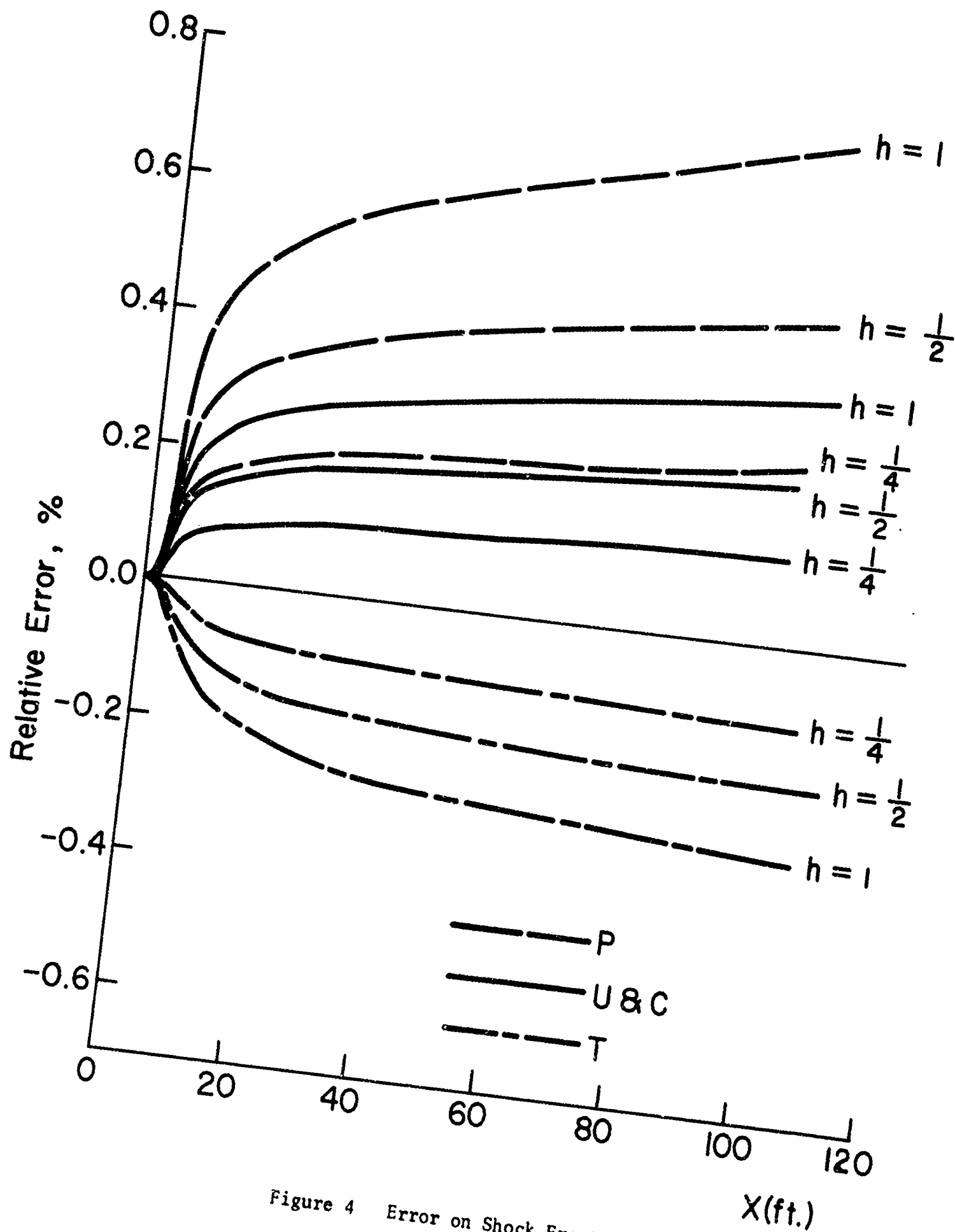


Figure 4 Error on Shock Front

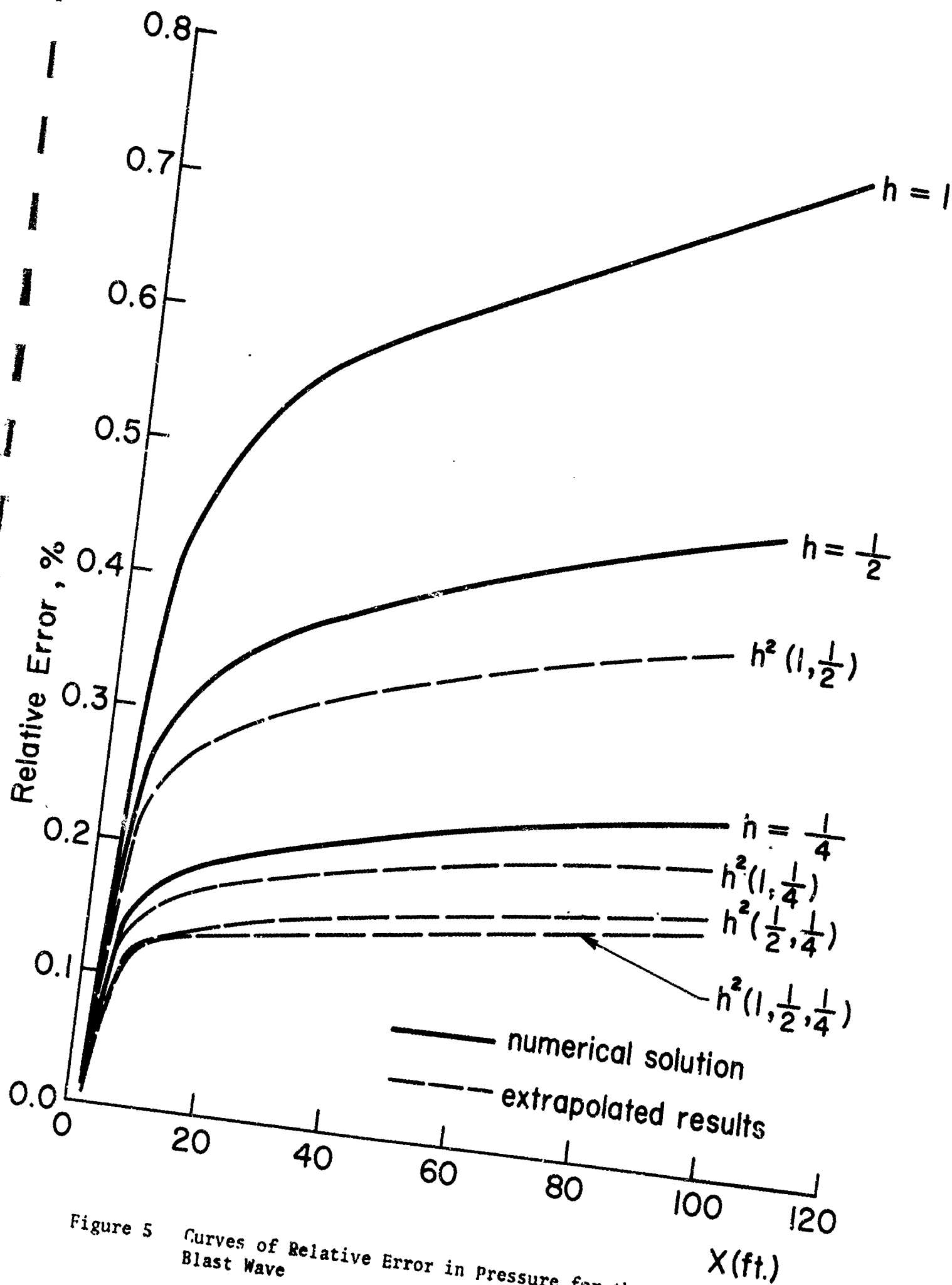


Figure 5 Curves of Relative Error in Pressure for the plane Blast Wave

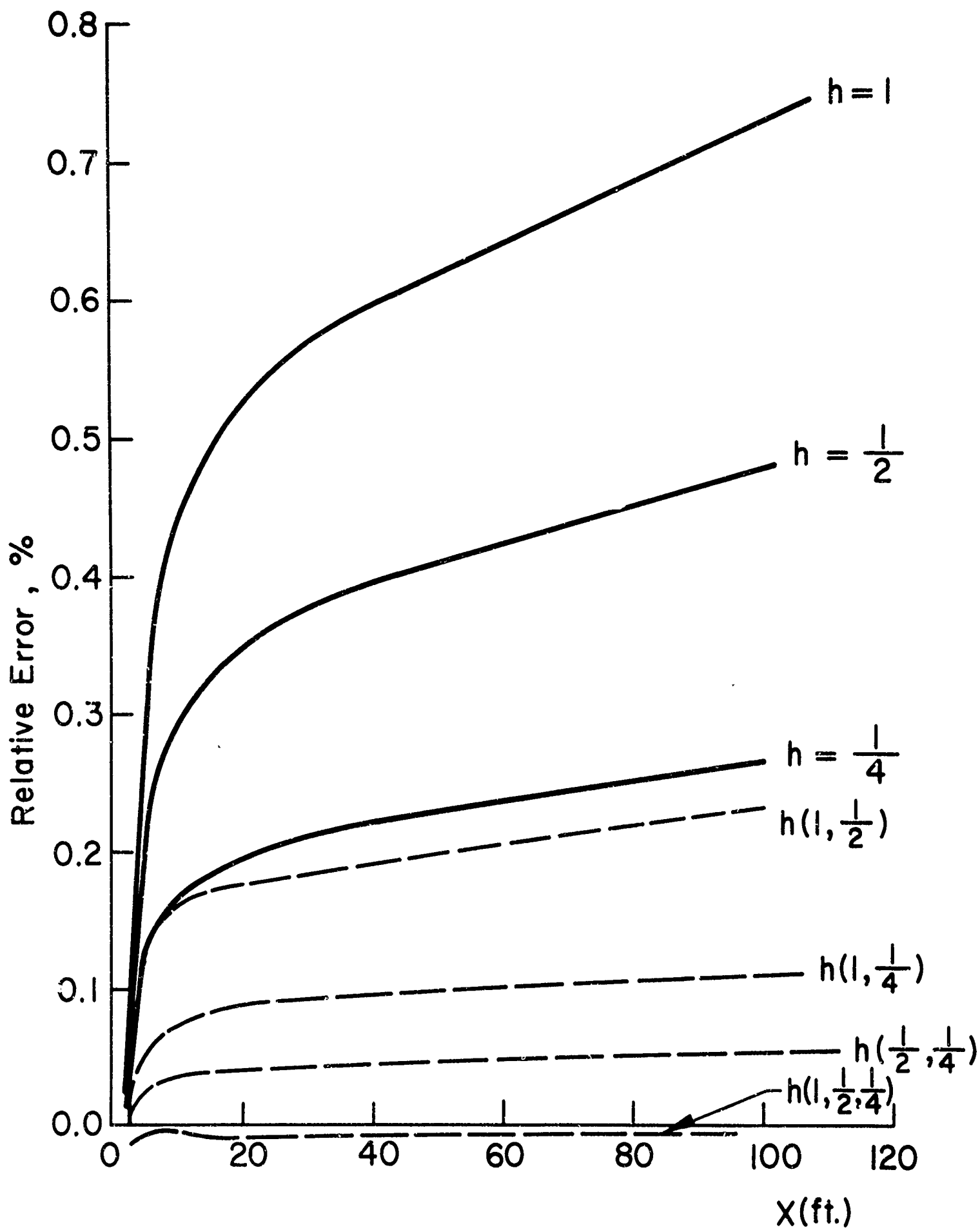


Figure 6 Curves of Relative Error in Pressure for the Plane Blast Wave

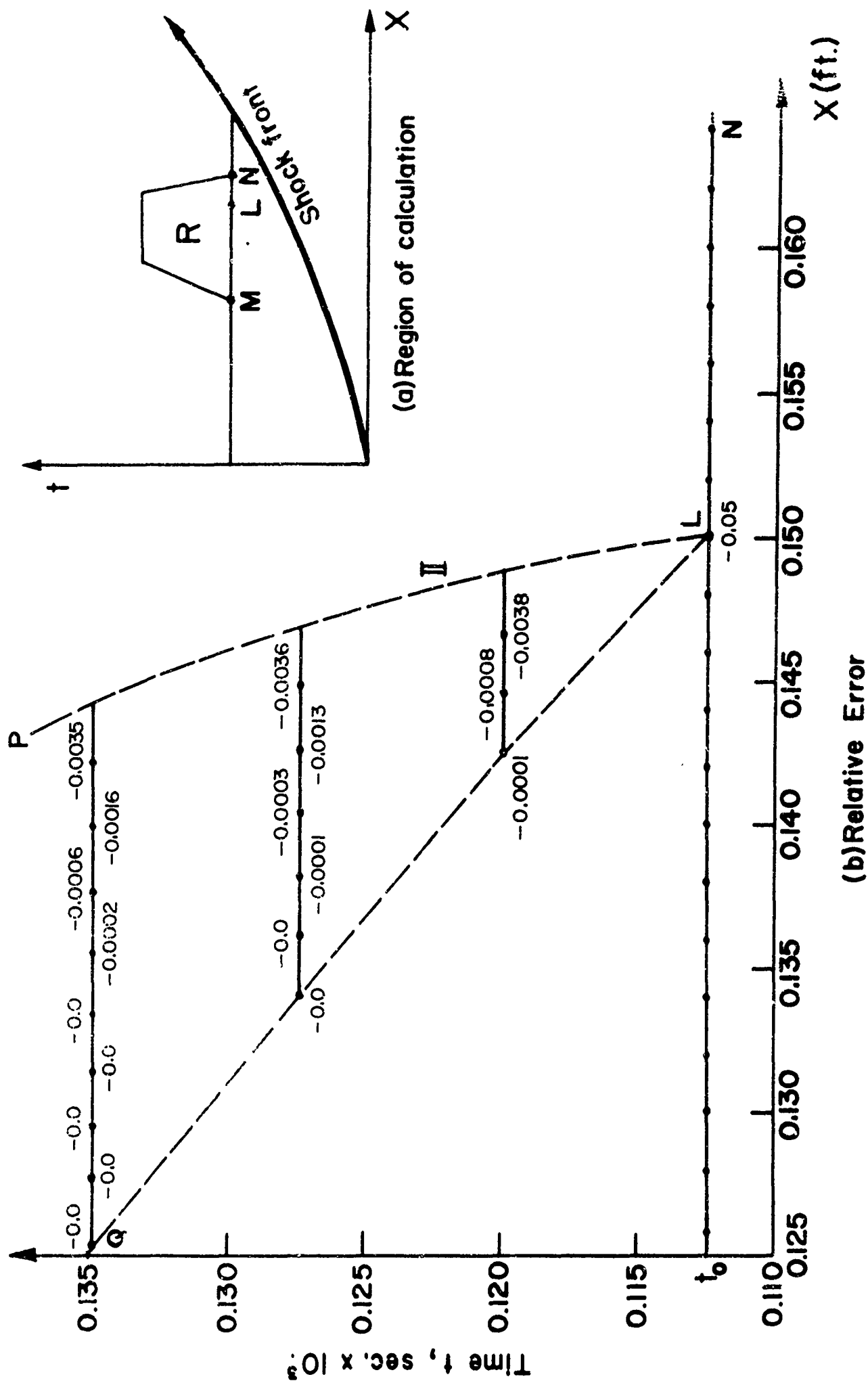


Figure 7 Physical Plane Showing Widening of Range of Influence and Relative Errors

APPENDIX A

Iteration Procedure

In this appendix, the detailed iteration procedure for the constant time method is described.

The basic step in the solution of the unsteady one-dimensional fluid flow problem by the constant time method involves the determination of all flow variables u , c , p , x at a set of discrete points along a constant time line, when those variables are known along a previous time line. As it was mentioned before the choice of Δt is subject to the condition $\Delta t < \Delta x/c$ for all interior points, except points on the back boundary and those on the shock front. For this particular problem we are fortunate to be able to determine the proper Δt based on properties near the back boundary without searching through the complete initial time line. More specifically, the criterion used (ref. Fig. A 1) is,

$$\Delta t = R \frac{x_3 - x_2}{c_2}, \text{ where } R = 0.99$$

Repeated application of the above step, with proper boundary conditions, yields solution of the problem. For the blast wave problem, we encounter four basic types of points. They are:

1. Points on the back boundary
2. Interior points which lie within the region of numerical solution
3. Points on the shock front
4. Points on added path line

Basic unit operations used in solving these four different types of points are described as follows:

1. Points on the Back Boundary

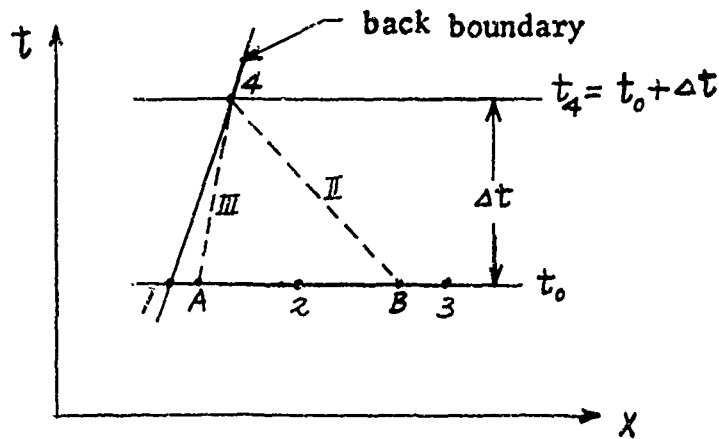


Figure A.1 Physical Plane

Referring to Fig. A.1, all variables (u , c , p , x , t) at points 1, 2, and 3 are known, the variables at point 4 are to be determined. Points 1 and 4 are on the back boundary. Points 4 and A are on the same III characteristics while points B and 4 are joined by a common II-characteristic. Point 4 must satisfy the equation of the back boundary which is

$$x_4 = k \left(\frac{E}{\rho_1} \right)^{\frac{1}{3}} t_4^{\frac{2}{3}} \quad (\text{A.1})$$

where k = constant (the position ratio), and as discussed earlier, the particle velocity along the back boundary is prescribed by the exact solution

$$u_4 = F \frac{4}{3(r+1)} \left(\frac{E}{\rho_1 t_4} \right)^{\frac{1}{3}} \quad (\text{A.2})$$

where $F = \text{constant}$, (the velocity ratio u_b/u_s). Since Δt has been determined, t_4 , the time at point 4, is known to be $t_0 + \Delta t$. Therefore x_4 and u_4 can be calculated readily from equations (A.1) and (A.2), respectively. The finite difference forms of the physical characteristic equations are:

$$\text{Along II} \quad \frac{x_4 - x_B}{\Delta t} = \bar{u}_{B4} - \bar{c}_{B4} \quad (\text{A.3})$$

$$\text{Along III} \quad \frac{x_4 - x_A}{\Delta t} = \bar{u}_{A4} \quad (\text{A.4})$$

where the barred quantities represent the average values, i.e.,

$$\bar{u}_{B4} = \frac{u_B + u_4}{2}$$

The corresponding state characteristic equations are:

$$\text{Along II} \quad \frac{p_4 - p_B}{u_4 - u_B} = r \frac{\bar{p}_{B4}}{\bar{c}_{B4}} \quad (\text{A.5})$$

$$\text{Along III} \quad \frac{c_4}{c_A} = \left(\frac{p_4}{p_A} \right)^{\frac{r-1}{2r}} \quad (\text{A.6})$$

The properties at points A and B are interpolated from the known values at points 1 and 2, or 2 and 3. Since u is very much linear with respect to x throughout the region, linear interpolation formulas are used to determine u at points A and B.

$$u_A = u_1 + \frac{x_A - x_1}{x_2 - x_1} (u_2 - u_1) = U_A(x_A) \quad (\text{A.7})$$

$$u_B = u_2 + \frac{x_B - x_2}{x_3 - x_2} (u_3 - u_2) = U_B(x_B) \quad (\text{A.8})$$

For the other properties, c and p , at points A and B, quadratic interpolation formulas are applied.

$$C_B = A_c(x_B - x_2)^2 + B_c(x_B - x_2) + C_2 = C(x_B) \quad (A.9)$$

$$P_B = A_p(x_B - x_2)^2 + B_p(x_B - x_2) + P_2 = P(x_B) \quad (A.10)$$

$$C_A = A_c(x_A - x_2)^2 + B_c(x_A - x_2) + C_2 = C(x_A) \quad (A.11)$$

$$P_A = A_p(x_A - x_2)^2 + B_p(x_A - x_2) + P_2 = P(x_A) \quad (A.12)$$

where

$$A_c = \frac{\begin{vmatrix} C_1 - C_2 & x_1 - x_2 \\ C_3 - C_2 & x_3 - x_2 \end{vmatrix}}{DET} \quad DET = \begin{vmatrix} (x_2 - x_1)^2 & x_1 - x_2 \\ (x_3 - x_2)^2 & x_3 - x_2 \end{vmatrix}$$

$$B_c = \frac{\begin{vmatrix} (x_2 - x_1)^2 & C_1 - C_2 \\ (x_3 - x_2)^2 & C_3 - C_2 \end{vmatrix}}{DET}$$

$$A_p = \frac{\begin{vmatrix} P_1 - P_2 & x_1 - x_2 \\ P_3 - P_2 & x_3 - x_2 \end{vmatrix}}{DET}$$

$$B_p = \frac{\begin{vmatrix} (x_2 - x_1)^2 & P_1 - P_2 \\ (x_3 - x_2)^2 & P_3 - P_2 \end{vmatrix}}{DET}$$

The ten equations (A.3) to (A.12) can be solved for the ten unknowns x_A , x_B , u_A , u_B , c_A , c_B , p_A , p_B , c_4 , and p_4 . The solution is accomplished by the following iteration scheme. To avoid possible confusion in notations in the description of the iteration procedures, a convention in using the superscripts will be adopted throughout this appendix unless otherwise specified. Superscript o , on any variables, such as u_A^o , represents the initial estimation of the value at point A. A single

prime on any variable, such as u_B' , designates the first approximation at point B. Similarly a double prime on any variable indicates the second approximation of the value of such a variable, etc.

First Iteration

Rewriting equations (A.3) to (A.12) in the above mentioned notation and making some approximations,

$$x_B' = x_4 - \left(\frac{u_A^0 + u_4}{2} - \frac{c_B^0 + c_4^0}{2} \right) \Delta t \quad (\text{A.13})$$

$$x_A' = x_4 - \left(\frac{u_A^0 + u_4}{2} \right) \Delta t \quad (\text{A.14})$$

$$u_A' = U_A(x_A') \quad (\text{A.15})$$

$$u_B' = U_B(x_B') \quad (\text{A.16})$$

$$c_B' = C(x_B') \quad (\text{A.17})$$

$$P_B' = P(x_B') \quad (\text{A.18})$$

$$c_A' = C(x_A') \quad (\text{A.19})$$

$$P_A' = P(x_A') \quad (\text{A.20})$$

$$P_4' = \gamma \left(\frac{P_B' + P_4^0}{c_B' + c_4^0} \right) (u_4 - u_B') + P_B' \quad (\text{A.21})$$

$$c_4' = \left(\frac{P_4'}{P_A'} \right)^{\frac{\gamma-1}{2\gamma}} c_A' \quad (\text{A.22})$$

The iteration procedure is initiated by estimating values for u_B^0 , c_B^0 , c_4^0 , u_A^0 , and p_4^0 . It is reasonable to assume that

$$u_B^0 = u_A^0 = u_2, \quad c_B^0 = c_4^0 = c_2, \quad P_4^0 = \frac{P_1 + P_2}{2}$$

With these estimations, x_B^0 and x_A^0 can be calculated from (A.13) and (A.14), respectively. Once x_B^0 and x_A^0 are determined the properties u_A^0 , u_B^0 , c_B^0 , p_B^0 , c_A^0 , and p_A^0 can be computed from the interpolation formulas (A.15) to (A.20). Finally, from equations (A.21) and (A.22), p_4^0 and c_4^0 may be obtained independently. The first approximation of the properties at point 4 and at points A and B is now complete.

Second Iteration

Once the first approximation is accomplished, the second and subsequent approximations of the variable at point 4 can be established from a set of equations similar to equations (A.13) and (A.22). Repeating the procedure in the first iteration with the primed values replacing the estimated values (values with superscript "o") and double primed variables instead of single primed variables, we can obtain the second approximation. This iteration process may be continued until the desired degree of accuracy is reached. In the present calculation, the iteration was stopped when the pressure at point 4 converged to within .0005% of its value in the previous iteration. For most points in this problem, not more than four iterations were required to achieve the desired accuracy.

2. Interior Points

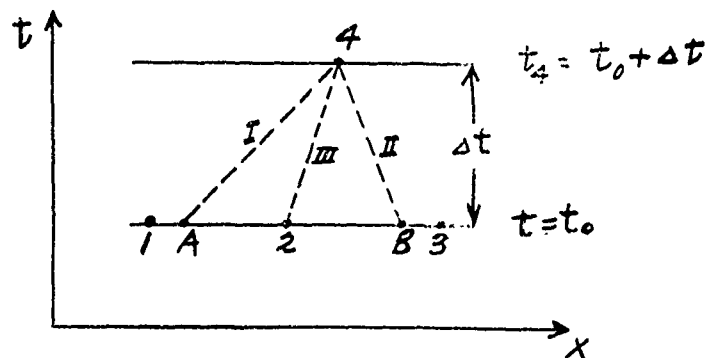


Figure A.2 Physical Plane

Referring to (A.2), all variables are assumed known at points 1, 2, and 3, and Δt is predetermined. The variables at point 4 are to be determined. Point A and point B lie on the characteristics along I and II issued from point 4, respectively, and points 4 and 2 are on the same path line (III-characteristic). Since Δt is chosen such that $\Delta t \leq \Delta x/c$, therefore for most interior points, point A lies between point 1 and 2, and point B between points 2 and 3. For interior points next to the back boundary or shock front some exceptional cases occur. These special cases are covered later in this section.

The finite difference forms of the physical characteristic equations are:

$$\text{Along I} \quad \frac{x_4 - x_A}{\Delta t} = \bar{u}_{A4} + \bar{c}_{A4} \quad (\text{A.23})$$

$$\text{Along II} \quad \frac{x_4 - x_B}{\Delta t} = \bar{u}_{B4} - \bar{c}_{B4} \quad (\text{A.24})$$

$$\text{Along III} \quad \frac{x_4 - x_2}{\Delta t} = \bar{u}_{24} \quad (\text{A.25})$$

The corresponding state characteristics are:

$$\text{Along I} \quad \frac{p_4 - p_A}{u_4 - u_A} = -\gamma \frac{\bar{p}_{A4}}{\bar{c}_{A4}} \quad (\text{A.26})$$

$$\text{Along II} \quad \frac{p_4 - p_B}{u_4 - u_B} = \gamma \frac{\bar{p}_{B4}}{\bar{c}_{B4}} \quad (\text{A.27})$$

$$\text{Along III} \quad \left(\frac{c_4}{c_2}\right) = \left(\frac{p_4}{p_2}\right)^{\frac{\gamma-1}{2\gamma}} \quad (\text{A.28})$$

The following interpolation formulas for properties at points A and B are the same as defined before,

$$u_A = U_A(x_A) \quad (\text{A.29})$$

$$u_B = U_B(x_B) \quad (\text{A.30})$$

$$c_B = C(x_B) \quad (\text{A.31})$$

$$P_B = P(x_B) \quad (A.32)$$

$$C_A = C(x_A) \quad (A.33)$$

$$P_A = P(x_A) \quad (A.34)$$

From the twelve equations, (A.23) to (A.34), the twelve unknowns, $x_4, u_4, p_4, c_4, x_A, u_A, c_A, p_A, x_B, u_B, c_B, p_B$ can be solved. The approximate solution is obtained by the following iteration procedure.

First Iteration

The first approximation was begun by rewriting the system of equations (A.23) to (A.34) in a new form which involved some approximation.

We have

$$x_4' = x_2 + \left(\frac{u_2 + u_4^0}{2} \right) \Delta t \quad (A.35)$$

$$x_A' = x_4' - \left(\frac{u_A^0 + u_4^0}{2} + \frac{c_A^0 + c_4^0}{2} \right) \Delta t \quad (A.36)$$

$$x_B' = x_4' - \left(\frac{u_B^0 + u_4^0}{2} - \frac{c_B^0 + c_4^0}{2} \right) \Delta t \quad (A.37)$$

$$u_A' = U_A(x_A') \quad (A.38)$$

$$u_B' = U_B(x_B') \quad (A.39)$$

$$c_B' = C(x_B') \quad (A.40)$$

$$p_B' = P(x_B') \quad (A.41)$$

$$c_A' = C(x_A') \quad (A.42)$$

$$p_A' = P(x_A') \quad (A.43)$$

$$\frac{p_4' - p_A'}{u_4' - u_A'} = -\gamma \left(\frac{p_A' + p_4^0}{c_A' + c_4^0} \right) \quad (A.44)$$

$$\frac{p_4' - p_B'}{u_4' - u_B'} = \gamma \left(\frac{p_B' + p_4^0}{c_B' + c_4^0} \right) \quad (A.45)$$

$$c_4' = \left(\frac{p_4'}{p_2} \right)^{\frac{\gamma-1}{2\gamma}} c_2 \quad (A.46)$$

and x_4^0 is determined by assuming $u_4^0 = u_2$. Now x_A^0 and x_B^0 may be calculated from equations (A.36) and (A.37) with the following estimated values: $u_A^0 = u_1$, $u_B^0 = u_3$, $c_A^0 = c_1$, $c_B^0 = c_3$, $c_4^0 = c_2$. Substituting x_A^0 in equations (A.38), (A.42), and (A.43), the properties at point A, u_A^0 , c_A^0 , and p_A^0 , can be determined, and substituting x_B^0 in equations (A.39), (A.40), and (A.41) determines u_B^0 , c_B^0 , and p_B^0 . Setting p_4^0 equal to p_2 , equations (A.44) and (A.45) can be solved simultaneously for the properties p_4^0 and u_4^0 . Finally c_4^0 is computed from equation (A.46). The first approximation of the properties at point 4 and points A and B are now complete.

Second Iteration

By repeating the above procedures in the first iteration with single primed values replacing the estimated values and double primed variables instead of single primed variables, the second approximation can be established. The iteration process may be continued until the desired degree of accuracy is reached. In the present calculation, the iteration was stopped when the properties u_4 and p_4 are both converged to within .0005% of their corresponding values in the previous iteration.

Special Cases

Case A (Figure A.2a)

Point 4 lies on the left of the back boundary. Since it lies outside the region of our solution, computation for such points are skipped.

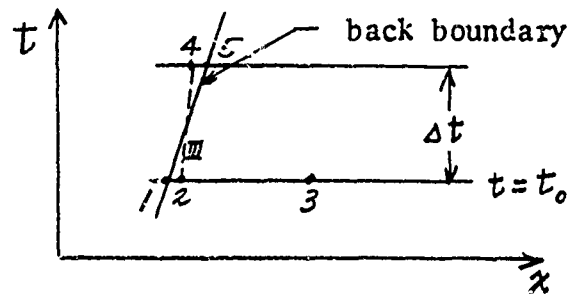


Figure A.2a Physical Plane

Case B (Figure A.2b)

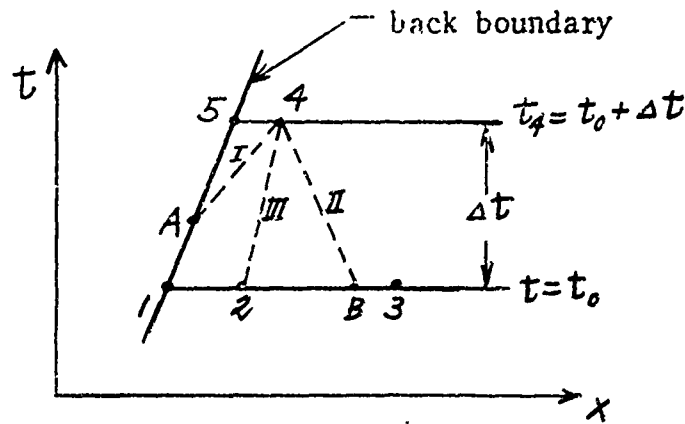


Figure A.2b Physical Plane

The I-characteristic issued from point 4 intersecting the back boundary at A before reaching $t = t_0$. The general iteration procedure for the interior point was followed except modifications were made to determine the location of point A and the properties at A. Here the local back boundary between points 1 and 5 was approximated by a straight line

$$\frac{x_A - x_1}{t_1 - t_0} = \frac{x_5 - x_1}{\Delta t} \quad (\text{A.47})$$

The finite-difference form of the I-characteristic equation in the physical plane is

$$\frac{x_4 - x_A}{t_4 - t_A} = \frac{u_4 + u_A}{2} + \frac{c_4 + c_A}{2} \quad (\text{A.48})$$

After x_4^0 is determined by equation (A.35), the first approximation of the location A, x_A^0 can be solved from the following version of equations (A.47) and (A.48) (i.e., equations (A.49) and (A.50)) with the indicated estimated values, bearing in mind that point 5 is a known point by this time.

$$\frac{x_A^0 - x_1}{t_A^0 - t_0} = \frac{x_5 - x_1}{\Delta t} \quad (\text{A.49})$$

$$\frac{x_4^0 - x_A^0}{t_4 - t_A} = \frac{u_4^0 + u_A^0}{2} + \frac{c_A^0 + c_4^0}{2} \quad \left(\begin{array}{l} \text{... } c_4^0 = c_2, u_A^0 = u_1 \\ \text{and } c_A^0 = c_1 \end{array} \right) \quad (\text{A.50})$$

The properties at point A were determined by linear interpolation formulas

$$u_A' = u_1 + \left(\frac{x_A' - x_1}{x_5 - x_1} \right) (u_5 - u_1)$$

$$c_A' = c_1 + \left(\frac{x_A' - x_1}{x_5 - x_1} \right) (c_5 - c_1)$$

$$p_A' = p_1 + \left(\frac{x_A' - x_1}{x_5 - x_1} \right) (p_5 - p_1)$$

Case C (Figure A.2c)

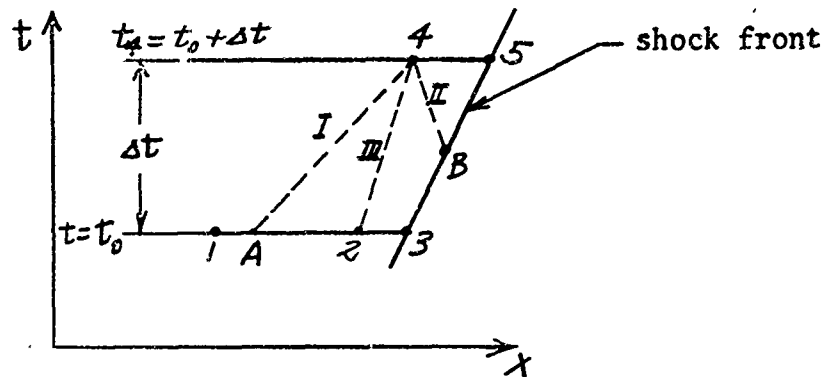


Figure A.2c Physical Plane

The II-characteristic intersects the shock front before it reaches the line $t = t_0$. Points next to the shock front are always computed after the properties at the shock front (point 5) has been determined. The actual procedure in establishing the properties at point 5 will be described later. Here we consider x_5 , t_5 , u_5 , c_5 , and p_5 to be known values. The modification in the iteration procedure for this case is similar to that of special case B. After x_4^0 is determined from equation (A.35), and assuming that the shock front between points 3 and 5 to be a straight line, x_B^0 can be solved from the equations,

$$\frac{x_B' - x_3}{t_B - t_0} = \frac{x_5 - x_3}{\Delta t}$$

$$\frac{x_4' - x_B'}{t_4 - t_B} = \left(\frac{u_4^0 + u_B^0}{2} - \frac{c_4^0 + c_B^0}{2} \right) \quad \left(\begin{array}{l} \text{set } c_4^0 = c_2 \quad u_B^0 = u_3 \\ \text{and } c_B^0 = c_3 \end{array} \right)$$

The linear interpolation formulas, which are used to determine the properties (first approximation) at point B, are

$$u_B' = u_3 + \left(\frac{x_B' - x_3}{x_5 - x_3} \right) (u_5 - u_3)$$

$$c_B' = c_3 + \left(\frac{x_B' - x_3}{x_5 - x_3} \right) (c_5 - c_3)$$

$$p_B' = p_3 + \left(\frac{x_B' - x_3}{x_5 - x_3} \right) (p_5 - p_3)$$

3. Points on the Shock Front

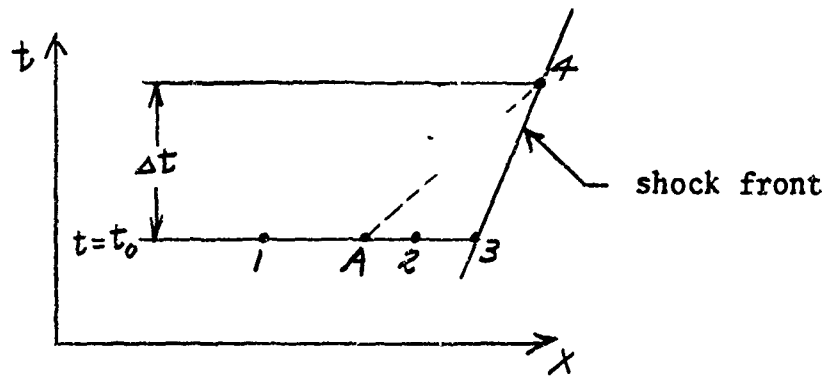


Figure A.3 Physical Plane

From the definition of shock wave velocity

$$\left(\frac{dx}{dt} \right)_{\text{shock}} = U = \frac{\gamma + 1}{2} \bar{u}$$

we have

$$\frac{x_4 - x_3}{\Delta t} = \bar{u}_{34} \quad (\text{A.51})$$

The finite-difference form of the I-physical and state characteristics are,

$$\frac{x_4 - x_A}{\Delta t} = \bar{u}_{A4} + \bar{c}_{A4} \quad (\text{A.52})$$

$$\frac{p_4 - p_A}{u_4 - u_A} = -\gamma \frac{\bar{p}_{A4}}{\bar{c}_{A4}} \quad (\text{A.53})$$

The properties at point 4 must satisfy the strong shock equations. Therefore

$$u_4 = \sqrt{\frac{2 p_4}{\rho_1 (\gamma + 1)}} \quad (\text{A.54})$$

$$c_4 = \sqrt{\frac{\gamma (\gamma - 1)}{2}} u_4 \quad (\text{A.55})$$

Linear interpolation formulas are used to determine u and c at point A. Since in this case point A may lie anywhere between point 1 and point 3, two sets of formulas (A.56a), (A.57a), and (A.56b), (A.57b) are provided. If A lies between points 1 and 2

$$u_A = u_1 + \left(\frac{x_A - x_1}{x_2 - x_1} \right) (u_2 - u_1) = U_1(x_A) \quad (\text{A.56a})$$

$$c_A = c_1 + \left(\frac{x_A - x_1}{x_2 - x_1} \right) (c_2 - c_1) = C_1(x_A) \quad (\text{A.57a})$$

or if A lies between points 2 and 3

$$u_A = u_2 + \left(\frac{x_A - x_2}{x_3 - x_2} \right) (u_3 - u_2) = U_2(x_A) \quad (\text{A.56b})$$

$$c_A = c_2 + \left(\frac{x_A - x_2}{x_3 - x_2} \right) (c_3 - c_2) = C_2(x_A) \quad (\text{A.57b})$$

Since p varies rapidly with respect to x near the shock front, the quadratic interpolation formula is applied. From equation (A.12)

$$p_A = A_p(x_A - x_2)^2 + B_p(x_A - x_2) + p_2 = P(x_A) \quad (\text{A.58})$$

From the eight equations, (A.51) to (A.58), the eight unknowns x_4 , u_4 , c_4 , p_4 , x_A , u_A , c_A , and p_A may be determined. The solution is accomplished by the following iteration procedure.

First Iteration

Equations used in the first approximation are:

$$x_4' = x_3 + \left(\frac{u_3 + u_4^0}{2} \right) \Delta t \quad (\text{A.59})$$

$$x_A' = x_4' - \left(\frac{u_A^0 + u_4^0}{2} + \frac{c_A^0 + c_4^0}{2} \right) \Delta t \quad (\text{A.60})$$

$$u_A' = U_1(x_A') \quad \text{or} \quad U_2(x_A') \quad (\text{A.61})$$

$$c_A' = C_1(x_A') \quad \text{or} \quad C_2(x_A') \quad (\text{A.62})$$

$$p_A' = P(x_A') \quad (\text{A.63})$$

$$P_4' = P_A' - r \left(\frac{P_A' + P_4^0}{C_A' + C_A^0} \right) (u_4^0 - u_A') \quad (A.64)$$

$$u_4' = \sqrt{\frac{2 P_4'}{r_1 (r+1)}} \quad (A.65)$$

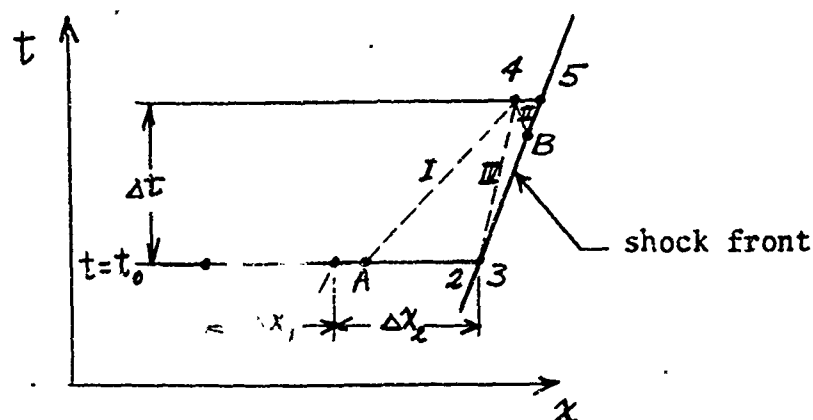
$$C_4' = \sqrt{\frac{r(r-1)}{2}} u_4' \quad (A.66)$$

where equations (A.61) and (A.62) are the appropriate linear interpolation formulas. The first iteration is performed with the following initial-input values: $u_4^0 = u_3$, $c_4^0 = c_3$, $p_4^0 = p_3$, $u_A^0 = u_2$, and $c_A^0 = c_2$. The first approximation of all the eight variables can be computed one by one from equations (A.59) to (A.66).

Second Iteration

The second iteration is accomplished by replacing the initial input, u_4^0 , c_4^0 , and p_4^0 (estimated values), with the average values of the initial input and the first approximation of the corresponding variable, u_A^0 by u_A^1 , c_A^0 by c_A^1 , and replacing single primed variables with double primed variables in equations (A.59) to (A.66), and computing the eight variables following the sequence of equations. Repeating the second iteration using the average values of the first approximation and the second approximation as inputs and substituting the double primed variables with triple primed variables, we can obtain the third approximation. The iteration procedure may continue until the desired accuracy is reached. In this case, the iteration was stopped when the pressure at point 4 converged to within .0002% of its value in the last iteration. The reason for using the average values of the initial input and the current approximation as the new input in the next iteration rather than just the current approximation of the corresponding variables is because the later scheme results diverge in subsequent iterations.

4. Points on Added Path Line



Referring to Fig. A.4, when $\Delta x_2 \geq \Delta x_1$, a new particle path will be initiated from the shock front for the purpose of keeping the data points populated more or less uniformly throughout the region of calculation. As mentioned in special case C point 4 is always treated after point 5 has been established. The present case is exactly the same as in special case C except point 2 and point 3 coincide. Hence, the same iteration procedure can be used to determine the properties at point 4.

*Instead of computing p_u^0 and u_u^0 separately from (A.64) and (A.65), we could solve the following two equations simultaneously for p_u^0 and u_u^0 .

$$\frac{P_4' - P_A'}{u_4' - u_A'} = -\gamma \left(\frac{P_A' + P_4^0}{C_A' + C_4^0} \right) \quad \text{and}$$

$$\frac{P_4'}{u_4'} = \frac{P_4^0}{\sqrt{\frac{2P_4^0}{\rho_1(\gamma+1)}}}$$

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13. ABSTRACT <p>The numerical method of characteristics with constant time grids is applied to the plane blast wave problem. The calculation begins at a constant time line bounded by the shock front and a fixed back boundary which lies halfway between the shock front and the time axis. Along the initial time line, all three flow variables are prescribed and on the back boundary, the local flow velocity is prescribed. The prescribed values are calculated from the similarity solution. For the shock front, the strong shock equations are applied.</p> <p>When compared to the exact similarity solution, the results of this method are found to be accurate to within .5% for all variables throughout the region of calculation, after a pressure drop of 98%. Both h^2-type and h-type extrapolations of calculated results improve the accuracy; however, h-type extrapolation is more favorable.</p> <p>The effects on the domain of dependence and range of influence are studied briefly.</p>			

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Blast Wave Similarity Solution Method of Characteristics Constant Time Scheme Domain of Dependence Range of Influence Error Analysis						

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